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PROBABILISTIC ELASTIC SHAKEDOWN RESPONSE AND LIMIT LOAD ANALYSES FOR 90° BACK-TO-BACK PIPE BENDS USING DEEP LEARNING–BASED DIRECT NUMERICAL ANALYSIS FRAMEWORK

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ABSTRACT

A 90° back-to-back pipe configuration, characterized by consecutive bends, is widely employed in industrial applications such as heat exchangers and energy systems due to its advantages in spatial efficiency and flow continuity. However, these structures face significant reliability challenges. Stress concentrations caused by geometric curvature create vulnerable points in the structure. Additionally, damage accumulation under cyclic loading and uncertainties in both manufacturing and operational processes further complicate reliable performance prediction. This study proposes an integrated probabilistic analysis framework for quantitatively evaluating the structural responses related to elastic shakedown limits and plastic limit loads, while accounting for uncertainties in geometry, material properties, and loading conditions. The analysis is based on the Linear Matching Method (LMM), implemented through a user-defined subroutine in ABAQUS. To reduce the computational cost associated with repeated simulations, a deep learning-based surrogate model was developed using data generated from key input variables. By employing the surrogate model, thousands of Monte Carlo simulations were efficiently performed to derive the probabilistic distribution of structural responses. Based on these results, the elastic shakedown-safe domain and the boundary conditions leading to plastic collapse were probabilistically defined. Furthermore, survival probability analysis was conducted to present load boundaries that satisfy performance requirements under uncertainty, providing a reliability-based perspective on structural safety.

This study offers a computationally efficient and accurate analysis framework that enables the probabilistic definition of allowable load boundaries, contributing significantly to the reliability-based design and evaluation of high-risk piping components under multiple sources of uncertainty.

KEYWORD

Pipe bend, Deep Neural Network, Probabilistic analysis, Linear Matching Method

INTRODUCTION

The 90° back-to-back pipe configuration is widely used in industrial systems such as heat exchangers and turbines due to its ability to change flow direction and optimize spatial efficiency. However, repeated high-pressure fluid loading induces stress concentration at the elbows, increasing the risk of structural failure.

In practice, uncertainties such as ovality, wall thickness variation, material property fluctuations, and variable loading conditions arise during manufacturing and operation. These factors lead to discrepancies between actual and predicted structural behavior, which are often overlooked in conventional deterministic evaluations.

To address this, the present study proposes an integrated numerical analysis framework combining an advanced direct numerical method known as the Linear Matching Method (LMM) and a Deep Neural Network (DNN)-based surrogate model to enable probabilistic assessment of elastic shakedown and plastic collapse limit behaviors under geometric and material uncertainty.

Figure 1: Schematic of a NPS 10 in. SCH 40 STD90° back-to-back pipe bends.

MATERIAL AND METHODOLOGY

This study focuses on a 90° back-to-back pipe, which was modeled using approximately 21,000 elements of which type is C3D20R in the Abaqus finite element software. The material properties used for the simulation is SUS304 stainless steel, modeled with an elastic–perfectly plastic (EPP) material behavior.

Three primary sources of uncertainty were considered in the analysis. First, geometric imperfections such as ovality and wall thickness variations (thinning and thickening) were defined as probabilistic variables based on ASME acceptance criteria. Second, the material properties, specifically Young's modulus and yield strength, were assigned probabilistic distributions using experimental data from previous studies. Third, the bending angle of the pipe was treated as a probabilistic variable to reflect uncertainties in the applied loading conditions.

The Linear Matching Method (LMM) was implemented through a user-defined material subroutine (UMAT) in Abaqus to evaluate the elastic shakedown and limit load responses. The method provides multipliers that indicate the maximum safe scaling of loads before failure occurs, which are then used to construct probabilistic loading boundaries.

To efficiently perform many simulations, a Deep Neural Network (DNN)-based surrogate model was developed using the LMM-generated dataset. The surrogate model was then used to conduct Monte Carlo simulations, enabling fast and accurate probabilistic analysis of the structural response under uncertainty.

RESULT AND DISCUSSION

Elastic shakedown analysis results

The DNN-based surrogate model for shakedown analysis was trained on 1,496 data samples and demonstrated high prediction accuracy, with R^2 values (over 0.99) indicating strong agreement with LMM results. Using this model, 20,000 Monte Carlo simulations were performed to estimate survival probabilities for various loading conditions. The results showed that the original pipe, designed deterministically, was located near the 30% survival probability boundary. This suggests that conventional designs may lack sufficient reliability under uncertain conditions. Furthermore, as the load ratio (R) decreased, indicating a more dominant bending load, the width of the probability density function (PDF) increased, reflecting the stronger influence of geometric and material uncertainties. Notably, when R dropped below 0.25, the PDF width began to narrow again, suggesting that under pressure-dominant conditions, structural response becomes less sensitive to uncertainty. This trend was consistently observed in both shakedown and limit load analyses.

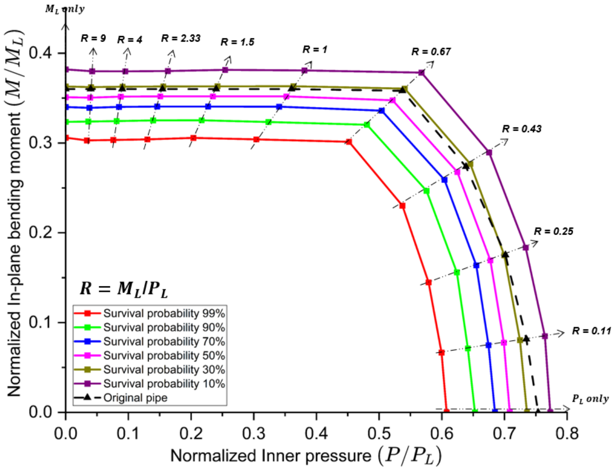

Figure 2: Probabilistic elastic shakedown boundary based on survival probability.

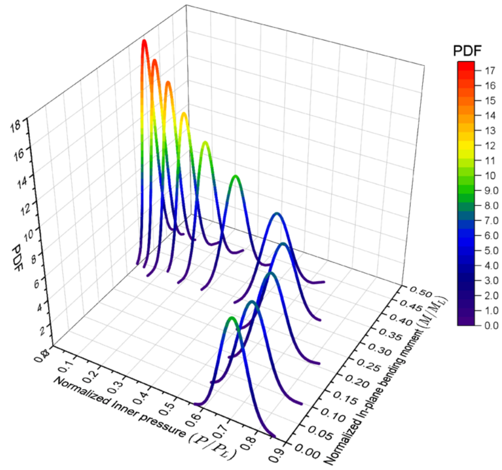

Figure 3: Probabilistic representation of elastic shakedown boundaries based on PDF distributions for each loading combination.

Limit load analysis results

For the limit load analysis, the surrogate model was trained on 1,023 data points and showed similarly high accuracy (over 0.99 R^2 value). A total of 30,000 Monte Carlo simulations were conducted to estimate the probabilistic limit load boundaries. As with the shakedown case, the original pipe was again positioned near the 30% survival probability boundary. The PDF distributions widened as the load ratio decreased, but began narrowing again when R dropped below 0.25, indicating a reduced influence of uncertainty under pressure-dominated loading. Additionally, the survival probability of the original pipe dropped from approximately 30% at R = 1.0 to about 20% at R = 0.43, indicating degraded structural reliability as bending loads become more dominant.

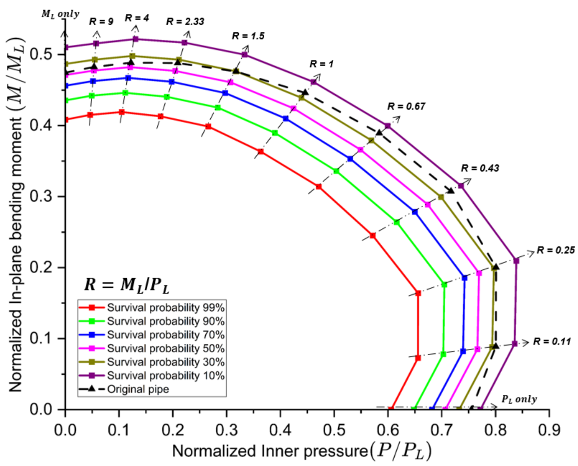

Figure 4: Probabilistic limit load boundary based on survival probability.

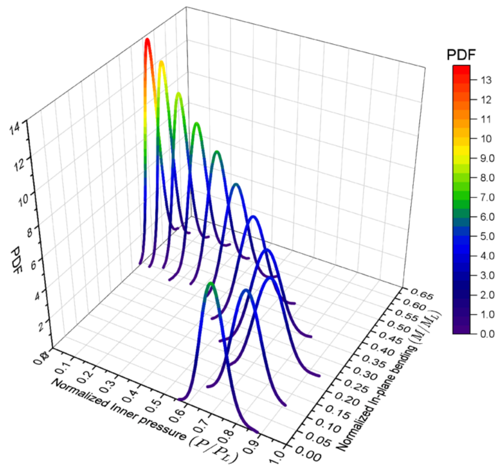

Figure 5: Probabilistic representation of limit load boundaries based on PDF distributions for each loading combination.

Reliability analysis

Based on the results of the surrogate models, reliability was assessed by evaluating the probability that the limit state function G(X) > 0 is satisfied. This was calculated as the proportion of Monte Carlo simulation results in which the predicted multiplier exceeded the design multiplier. The analysis revealed that certain load combinations achieved reliability levels above 90%, while others fell below 30%, identifying critical scenarios where structural safety is at risk. Notably, the variability in reliability was most significant at lower load ratios, where uncertainties in geometry and material properties had a stronger impact. This quantitative reliability assessment provides important insight into structural safety that cannot be captured by traditional deterministic approaches.

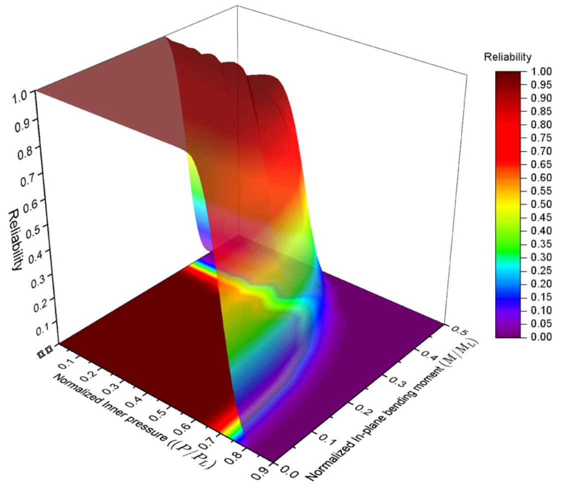

Figure 6: Elastic shakedown reliability for each loading combination based on limit state function.

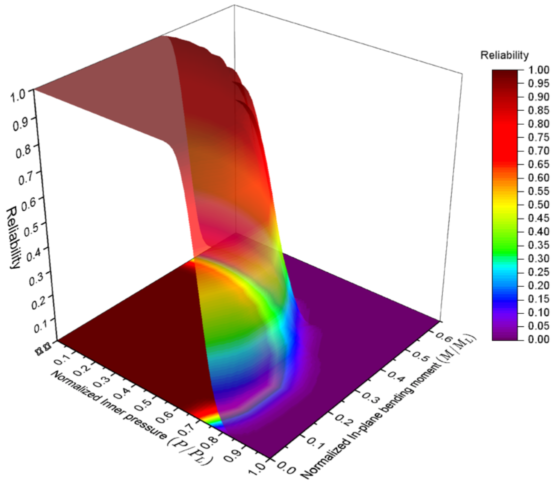

Figure 7: Limit load reliability for each loading combination based on limit state function.

Additionally, the proposed DNN-based surrogate model generated approximately 106,000 structural response datasets in just 17 days, achieving a computational time reduction of about 43 times compared to the estimated 736 days required for direct LMM simulations. This highlights the practical value of the proposed framework for large-scale probabilistic structural reliability assessments.

CONCLUSION

This study proposed an integrated numerical analysis framework for probabilistic evaluation of shakedown and limit load behavior in 90° back-to-back pipes under uncertainty. By combining LMM-based limit analysis with a DNN surrogate model, the framework achieved both accuracy and computational efficiency in structural reliability assessment.

The results demonstrated that the proposed method can effectively capture the effects of geometric, material, and loading uncertainties, and enable the establishment of more conservative and reliable design boundaries. This approach provides a practical alternative to conventional deterministic analysis and is particularly applicable to safety-critical piping systems operating under uncertain real-world conditions.

ACKNOWLEDGEMENT

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