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FOURIER SERIES TREND-BASED MODEL DEVELOPMENT FOR AZIMUTH PREDICTION

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ABSTRACT

Data analysis is pivotal in modelling the various trends that are primarily linear or nonlinear. Models enable professionals to identify the characteristics of a particular system and hence facilitate forecasting. In this work, a trend-based model was developed based on experimental data generated from Ekiti State of Nigeria by the use of theodolite for the prediction of solar azimuth orientation on a daily basis. Models enable researchers to carry out predicting and controlling functions based on previously conducted experiments and observations. The model has a maximum error of 4.72% and a correlation coefficient of 0.95 between the predicted and actual values. The modelling approach can help in scrutinizing complex and bogus systems at ease with the aid of computer programming. The applications of models in software development bring about a reduction in storage requirements since parameters are generated mathematically.

KEYWORD

Azimuth, Development, Forecasting Tools, Models, Trend

INTRODUCTION

Modelling is considered more as an art than a science and depends heavily on the experience and knowledge of the researchers involved. It requires both a good understanding of the nature of the process and a familiarity with the available models and methods. In some practical cases, the phenomena under study are of very high complexity, and any mathematical description is just a modest approximation. Unavoidable simplifications and approximations made during the modelling process can greatly alter the predicted behaviour of real-world phenomena. Applied mathematical modelling does not make sense without first defining the purpose of modelling. Before a model is developed, a specific existing problem should be described, and the possible implications and benefits of the model need to be explored. This application can be applied in solar tracking in setting photovoltaic panels perpendicularly via the consideration of azimuth angles. The azimuth angles are directly proportional to panel position for optimum radiation reception, hence the need to develop a model with continuous periodic occurrences leading to the adoption of the Fourier series model.

MATERIAL AND METHODOLOGY

The determination of Fourier series Sun Set Azimuth predictive models was accomplished by taking into consideration the movement of the sun about the earth. A theodolite made by Wild Company in Germany was used for monitoring the sun's movement in twelve months. A precision compass was also used to determine where to set up the theodolite tripod and levelling was done with spirit level while the orientations were determined with optical plumb bob telescope, lens, and the internal tilt meter of the instrument. The theodolite has a turning procedure to verify its level thereby avoiding azimuth errors. Readings were taken from 7 am to 6:30pm at thirty-minute intervals daily. Extracts of data collected from the experimental procedure were presented in Figure 1. The Figure was used to establish the daily trend following 7:00 am for the period of consideration. The mathematical models for predicting azimuth and altitude solar tracker positioning were derived from the recorded data whose graphs are presented Figure 1.

Development of a Fourier series (7.00 am) Start Azimuth Model

The azimuth and altitude angle obtained at 7.00 a.m. were plotted against the days to establish the trend daily in consideration of the year, while the subsequent years were considered as having similar periodic trends as presented in Figure 1.

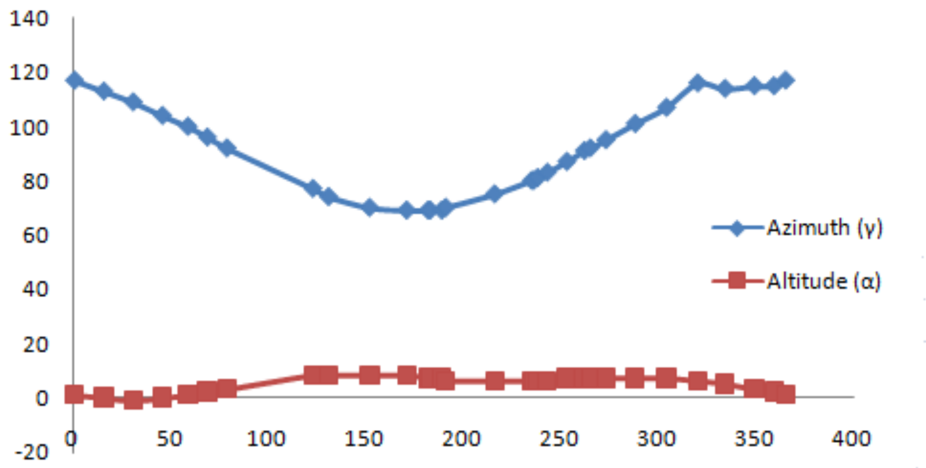

Figure 1: Azimuth and Altitude Angle for January 1 to December 2014 from Experiment.

Figure 1 shows the occurrence of the azimuth angle at 7am which is assumed as the average yearly starting angle of the sun from our geographical location at Ado-Ekiti, Nigeria (local time), it tends to replicate itself in a continuous trend on yearly basis. This trend for the azimuth angle could be represented by a cosine function as given in Equation 1.

$$f(x) = y = \int \cos x dx \tag{1}$$

$f(x)$ denotes function of the real variable x. This function is usually taken to be periodic, of period 2π, which is to say that $f(x+2\pi) = f(x)$, for all real number x. Such a function can be written as an infinite sum or series of simpler $2\pi -$periodic functions. Using an infinite sum of sine and cosine functions on the interval $(-\pi, \pi)$(Oladebeye and Ejiko, 2015).

For a periodic function $f(x)$that is integral on $(-\pi, \pi)$, the numbers $a_n = {}^1\!/_\pi \int_{-\pi}^{\pi} f(x)\cos(nx)dx$, $n \geq 0$ and $b_n = {}^1\!/_\pi \int_{-\pi}^{\pi} f(x)\sin(nx)dx$, $n \geq 1$ are called the Fourier coefficients of f. Also, by introducing the partial sums of the Fourier series for f often denoted by snf in Equation 2 is obtained.

$$(snf) = \frac{a_o}{2} + \sum_{n=1}^{N}[a_n \cos(nx) + b_n \sin(nx)], N \geq 0. \tag{2}$$

The partial sums for f are trigonometric polynomial. One expects that the function (snf) approximate the function f, and that the approximation improves as n tends to infinity. The infinite sum

$$\frac{a_o}{2}+\sum_{n=1}^{N}[a_n \cos(nx)+b_n \sin(nx)] \tag{3}$$

Is called the Fourier series of f

The Fourier series does not always converge, and even when it does converge for a specific value x_o of x, the sum of the series at x_o may differ from the value $f(x_o)$ of the function (Ejiko et. al, 2021a). According to Ejiko et. al., (2015) and Glyn et al., (1999) $f(x)$ is an even function because cosine is also an even function; therefore $f(-x)=f(x)$ and properties of even function b_n becomes 0. Using $f(x)$ as an even function as cosine trend Equation according to Dass (2009) and (2007).

$$f(x)=\frac{a_o}{2}+\sum_{n=i=1}^{a\leftrightarrow 0}(a_n \cos nx+b_n \sin x) \tag{4}$$

$$f(x)=\frac{a_0}{2}+\sum a_n \cos(nx) \tag{5}$$

$$\frac{a_0}{2}=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

Where $a_0/2$ serve as the amplitude function A

Therefore, A= $(a+c)/2$ (6)

From the graph in Fig. 1 the first period gives an Upper limit = 117 represent "a" and Lower limit = 69 represent "c"

where, the maximum point for the first periods is 'a' and minimum point is 'c'

A= $(117+69)/2=93$

And a_n as a function of the negative and positive quantity about the mid-point

$(a-c)/2=24$

Using the cosine trend Equation reported by Ejiko et. al. (2021b); Equation 7 is derived as a cosine Fourier series function.

$f(x)=\cos nx$ For a period of 2π

Therefore, 366 days = $2\pi; n$ cycles is 360o

366n = 2π this implies that n = $2\pi/366$ with this application the azimuth day model γ_D becomes

$$\gamma_D=\frac{a+c}{2}+\frac{a-c}{2}\sum_{x=1\rightarrow 366}^{\theta\rightarrow\infty}\cos\left(\frac{2\pi}{366}\right)^c x \tag{7}$$

Equation 8 is the Alpha-numeric model showing variables, a, c, and x, while Equation 9 gives the comprehensive numerical model for the daily start angle.

$$\gamma_S=93+24\sum_{x=1\rightarrow 366}^{\theta\rightarrow\infty}\cos(0.984x) \tag{8}$$

Where the Azimuth start angle of the day is γ_S, x is the day in the number of the year, or if we set the day in number of the year, D, to be equal to x, then Equation 8 is written as a function of D as shown in Equation 9.

$$\gamma_S = 93 + 24 \sum_{x=1 \to 366}^{\theta \to \infty} \cos(0.984D) \tag{9}$$

RESULT AND DISCUSSION

The maximum deviation between the predicted and actual azimuth angle was 17 degree. The ratio of maximum error with the full scale deflection was used to determine the maximum percentage azimuth error as captured in Equation 10.

$$\text{Maximum Percentage Azimuth error } \frac{Max(E_\gamma)}{FSD(360^o)} \times \frac{100}{1} = 4.72\% \tag{10}$$

The developed models were found to have a maximum error of 4.72% for azimuth angle and a correlation coefficient of 0.95 between the predicted and actual azimuth angles. The predicted thereby indicating a high degree of agreement between the actual data and the data predicted from the developed models.

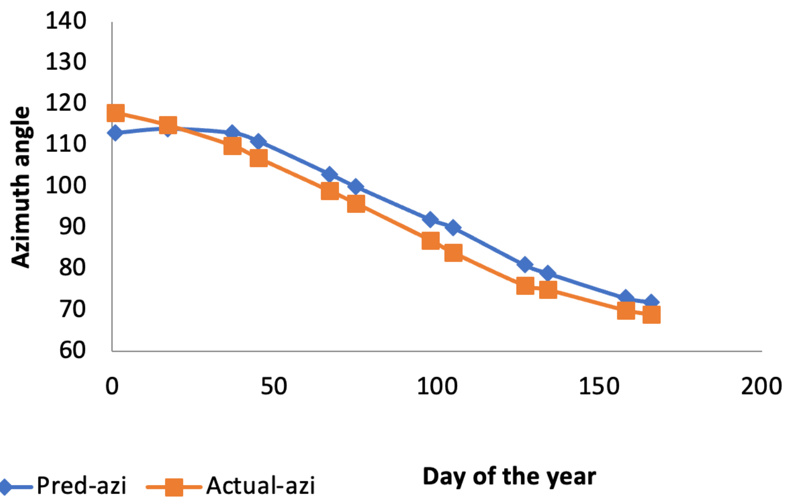

Figure 2: Differences between Predicted and Actual Azimuth Angles

CONCLUSION

Mathematical modelling is pivotal in the formulation of trend base models. These models enable the engineer to identify the characteristic of a particular system, condition, or situation. The effecting tracking of solar radiation can be actualized by the application of these models when programmed. The observed generated data when plotted graphical a trend evolved which is instrumental in the development of numerical models that can be used to rapidly predict azimuth on daily basis.

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